SECTION 4: FINANCE

4.1 SIMPLE INTEREST

Simple interest is calculated using the principle (*P)*, or original, amount of a loan or investment over a period of time. After a certain amount of time, we calculate the accumulated amount.

This include hire purchase agreements

$A = P(1 \pm ni)$

4.2 COMPOUND INTEREST

Compound interest is calculated using the principle(*P)*, or original, amount and the accumulated interest of that original amount over a period of time.

This is including inflation, reducing balance and population growth/shrinkage.

$A = P(1 \pm i)^n$

r= rate given as percentage or decimal

n= number of compounding periods

$i = \frac{r}{n}$

4.3 NOMINAL AND EFFECTIVE INTEREST

Nominal interest is the amount of interest you would have to pay in a year if the interest is only compounded once a year. But effective interest rate actually takes into account whether the interest is compounded daily, monthly, quarterly ect.

$$(1 + i_{eff}) = \left(1 + \frac{i_{nom}}{n}\right)^n$$

N is the number of times PER YEAR that interest is added

Daily	*n*=365
Monthly	*n*=12
Quarterly	*n*=4
Half-yearly (semi-annually)	*n*=2

4.4 FUTURE VALUE ANNUITIES

An annuity is simply regular payments made for a predetermined period of time.

This includes regular investments, sinking funds, savings plan, which we use this F formula for:

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

This also includes regular payments, loans, debt repayment which we use this P formula for:

$$P = \frac{x[1 - (1 - i)^{-n}]}{i}$$

4.5 OUTSANDING BALANCE OF LOAN (OB)

This is quite simply how much money you would still need to pay of an agreed loan at a given time.

let k= number of payments already made

$$OB = P = \frac{x\left[1-(1-i)^{-(n-k)}\right]}{i}$$

4.6 SPECIAL CASES

1.SINKING FUNDS, FUTURE VALUE ANNUITIES, INVESTMENTS

Generally, the payment is made at the end of the month (or other given time period). However sometimes, the question asks to set up the annuity when the payment is made at the beginning of the month. All you need to do is add an extra month's interest. i.e. $n+1$

$$F = \frac{x[(1+i)^{n+1}-1]}{i}$$

EXAMPLE 4.1:

If Sarah invests R5000 per month starting immediately for a period of 6 years. The interest rate is 7% p.a. What is the amount at the end of the investment period?

$$6years \times 12months = 72$$

$$F = \frac{x[(1+i)^{n+1}-1]}{i}$$

$$F = (5000)\frac{\left[\left(1+\frac{0.07}{12}\right)^{72+1}-1\right]}{\frac{0.07}{12}}$$

2. DEFFERRED LOANS- PAYMENTS START LATER ON

This occurs when for example a loan is granted but the first payment for the loan is only required in 3 months time. Normally loan payments start on the first of each month. Therefore, for the first 2 months the loan will be compounded and then you can use the normal loan formula.

EXAMPLE 4.2:

Peter borrows R18 000 but is only required to start repayments after 4 months. The loan needs to be paid over 2 years from the date that the loan is taken out. Calculate how much Peter needs to pay monthly (x) if the interest is 7% p.a.

$$P = 18000\left(1+\frac{0.07}{12}\right)^{\frac{3}{12}\times 12}$$

$$P = R18316{,}84\ldots$$

For the first 3 months the loan amount accumulates and at the beginning of the 4th month payment is made

*this is now the new value Peter needs to pay off.

$$18316{,}84 = x\left[\frac{1-\left(1+\frac{0.07}{12}\right)^{-21}}{\frac{0.07}{12}}\right]$$

For 2 years minus 3 month the loan is paid.
12months x 2= 24months
24 months - 3 months = 21 months

$$x = 929.28$$

4.7 PROBLEMS

1.How long will it take a motor car bought for R200 000 to depreciate to R50 000 at 16% p.a
(a) Using simple interest

$$A = P(1 \pm ni)$$

A great tip is to make a summary of the given information in the question

A=50 000
P=200 000
i=0.16 or 16%
n=?

$$50000 = 200000(1-n(0{,}16))$$
$$0{,}25 = 1-n(0{,}16)$$
$$n(0{,}16) = 0{,}75$$
$$n = 4{,}6875 \text{ years}$$

Therefore, it will take 4 years and 8 months (0,65 x12) to depreciate.

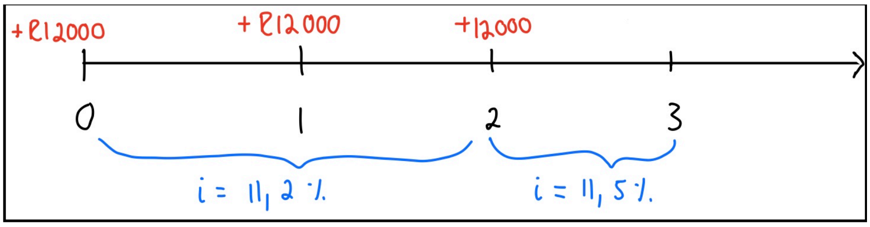
(b) Using compound interest

$$A = P(1 \pm i)^n$$

$$50000 = 200000(1-0{,}16)^n$$
$$0{,}25 = (1-0{,}16)^n$$
$$n = log_{(1-0{,}16)}0{,}25$$
$$n = 7{,}95 \text{ years}$$

Therefore, it will take 7 years and 11 months (0,95 x 12) to depreciate.

2. Emma wanted to save for an overseas trip. She makes a deposit of R12 000 now, another R12000 in one year's time and a further R12000 in two year's time. The trip will cost R50 000 after three years. If the interest rate of 11,2% compounded monthly is paid for the first 2 years and this rate rises to 11,5% for the rest of the time, what is the balance Emma will have to pay in three years?



Timeline example

For explanatory purposes I am going to split this question into 3 part for each sum of money added:

The first R12000 added is invested for 2 years at 11,2% and then for 1 year at 11,5% :

$$F_1 = 12000\left(1 + \frac{0{,}112}{12}\right)^{2\times12} \times \left(1 + \frac{0{,}115}{12}\right)^{12\times1}$$

The second R12000 added after 1 year is invested for 1 year at 11,2% and 1-year are 11,5 %

$$F_2 = 12000\left(1 + \frac{0{,}112}{12}\right)^{1\times12} \times \left(1 + \frac{0{,}115}{12}\right)^{12\times1}$$

The third R12000 is added after 2 years and is invested for 1 year at 11,5%

$$F_2 = 12000\left(1 + \frac{0{,}115}{12}\right)^{12\times1}$$

If we add F1, F2 and F3 together

$F = 43687{,}24$

Therefore, there is $R50000 - R43687{,}24 = R6312{,}76$ left for Emma to pay

3. Sasol buys equipment for 1,8 million rand.
(a) Calculate the quoted value of the equipment at the end of eight years if the value depreciates at a rate of 13,5% p.a on the compounded interest.

$A = P(1 \pm i)^n$

$A = 1800000(1 - 0.135)^8$

A= R 564 158,80

A=?
P=1800000
i=0.135 or 13,5%

(b) Calculate the expected cost if inflation is estimated to be 5,5% p.a.

$A = P(1 \pm i)^n$

$A = 1800000(1 + 0.055)^8$

A= R276 2435,22

A=?
P=1800000
i=0,055 or 5,5%
n

4. Calculate the effective interest rate if the nominal rate is 10,5% p.a. calculated daily.

$$(1 + i_{eff}) = \left(1 + \frac{inom}{n}\right)^n$$

$$(1 + i_{eff}) = \left(1 + \frac{0,105}{365}\right)^{365}$$

$$(1 + i_{eff}) = \left(1 + \frac{0,105}{365}\right)^{365}$$

$i_{eff} = 0,11069$

Therefore 11,07%

4.Nicole buys a truck for R300 000. The truck depreciates in value at 30% p.a. on a reducing balance. New truck prices increase at 15% p.a.

(a) Find the value of Nicole's truck in 4 year's time.

$A = P(1 \pm i)^n$

$A = 300000(1 - 0,03)^4$

A= R72 030

(b) New truck price: $A = P(1 \pm i)^n$

$A = 300000(1 + 0,15)^4$

A= R 524 701,88

(b) She now sets up a sinking fund to replace her truck in 4 years time, she will also use her current truck as a trade-in. How much will she have to save monthly into the fund if interest is 9,5% p.a.(compounded monthly)

How much will Nicole have to Save?

$R524701,88 - R72030 = R452671,88$

F= 452 671,88
x=?
i=0,095 or 9,5%

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$452671,88 = x\frac{\left[\left(1 + \frac{0,095}{12}\right)^{4\times12} - 1\right]}{\frac{0,095}{12}}$$

x= R7 788,89

5. Robyn bought a small house. She paid a R102 000 deposit which is equivalent to 12% of the house's value. She will pay the remaining value by taking out a house loan with monthly payments over 20 years at 9% p.a.(compounded monthly).

(a) Find the initial cost of the house and the monthly payments Robyn needs to make.

(b) Let *x* be the initial cost of the house

$x(0{,}12) = 102000$

$x = 102000 \div 0{,}12$

x = R 850 000

P=850 000 — 102000= 748 000

x=?

i=0,09 or 9%

$$P = \frac{x[1-(1-i)^{-n}]}{i}$$

$$748000 = x\left[\frac{1-\left(1+\frac{0{,}09}{12}\right)^{-20\times12}}{\frac{0{,}09}{12}}\right]$$

x = R 6 729,95

(c) Find the outstanding balance on the loan after 85 payments

$$OB = P = \frac{x[1-(1-i)^{-(n-k)}]}{i}$$

$$OB = 6729{,}95\left[\frac{1-\left(1+\frac{0{,}09}{12}\right)^{-(20\times12-85)}}{\frac{0{,}09}{12}}\right]$$

OB= R 615 509,74

OB=?

x= 6729,95

i=0,09 or 9%

n=20 x 12

(d) If Robyn misses payments 86,87,88 and 89. She agrees to pay R8500 from the end of month 90. Calculate how long it will take to pay off the loan.

Robyn misses 4 payments. The loan amount accumulates for 4 months (compounded). We use the compound interest formula to calculate the new loan amount Robyn owes.

$$A = 615509{,}74\left(1+\frac{0{,}09}{12}\right)^{\frac{4}{12}\times12}$$

A= R 634 183,81

Now we use the normal loan equation and solve for n.

$$P = \frac{x[1-(1-i)^{-n\times12}]}{i}$$

$$634123{,}81 = 8500\left\{\frac{1-\left(1+\frac{0{,}09}{12}\right)^{-n\times12}}{\frac{0{,}09}{12}}\right\}$$

$$0{,}56 = 1-\left(1+\frac{0{,}09}{12}\right)^{-n\times12}$$

$$0,44 = \left(1 + \frac{0,09}{12}\right)^{-n \times 12}$$
$$-n \times 12 = log_{\left(1+\frac{0,09}{12}\right)} 0,44$$
$$-n \times 12 = -109,87$$

n= 110 months or 9,16 years